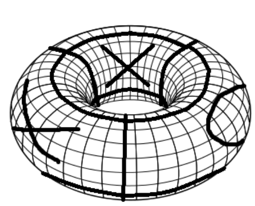
1 A Handful of Games

Here are a collection of games to play with two or more players. To win each game you will have to solve the secret behind it. For each game find out if there is a winning strategy and if so which player has it.

1. *Tic-Tac-Torus*. To play Tic-Tac-Toe on a Torus we use a special three by three grid. The edges on the top and bottom are glued together and the edges on the left and right are glued together. Which player has a winning strategy? Can you ever have a draw?

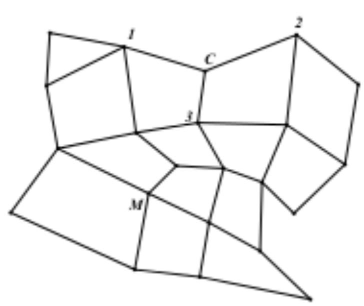


2. *Color the Grid*. You start with an $n \times m$ grid of graph paper. Players take turns coloring red one previously uncolored unit edge of the grid (including the boundary). A move is legal as long as no closed path has been created. If a player cannot make a legal move they lose. Which players has the winning strategy?

3. *One to Nine*. Between two players are the number one through nine, each written on a separate piece of paper. The players take turns taking one number from middle for themselves. A player wins if among the numbers they have collected there are exactly three which add to 15. For example a game might go like this:

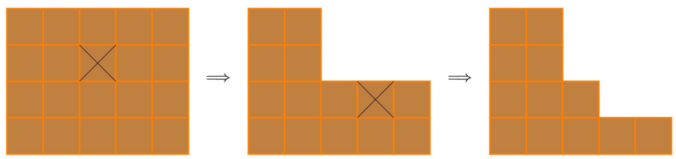
   P1 takes 2, P2 takes 8, P1 takes 6, P2 takes 7, P1 takes 5, P2 takes 3, then P1 takes 4 and wins because they have 4,5 and 6.

4. *Cat and Mouse*. A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.



Initially, the cat is at the point labeled C; the mouse is at M. The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. Can the cat win?

5. *Chomp.* A chocolate bar has n rows and m columns of chocolate. Players take turns taking a bite out of the chocolate bar. To take a bite they choose a square and then take every square above and to the right of the chosen square.



The square in the bottom left is poisoned. Whoever eats that square looses. Which player has the winning strategy? (Hint: you might be able to answer this question without actually discovering the strategy itself.)

6. *OutFactor.*Players take turns subtracting from a given number. If the current total is n, you may subtract any (integer) factor of n besides itself. You lose if this is impossible – that is, if the total is 1 at the start of your turn. Here is a sample game:

   The starting number is 12.

   Aino goes first, and she can subtract 1,2,3,4 or 6. She subtracts 4, leaving 8.

   Now Bruno can subtract 1,2 or 4. He subtracts 2, leaving 6.

   Aino subtracts 3, leaving 3.

   Bruno subtracts 1, leaving 1.

   Aino subtrats 1, leaving 1, and so Bruno loses.

   Suppose n is one more than an odd prime. Would you like to go first or second?